TRINITY COLLEGE

Mathematics Specialist Units 1,2
Test 6 2018

Proof, Complex Numbers

STUDENT'S NAME ____

DATE: Monday 17 September **TIME:** 50 minutes **MARKS**: 52

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, notes on one side of a single A4 page

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Express the recurring decimal 21.357575757….. as a rational number.

2. (6 marks)

(a) Given $(PQ)^3 = I$, show $QPQ = P^{-1}Q^{-1}P^{-1}$ where I is the identity matrix, P and Q are non-singular square matrices. [2]

(b) If matrix A is such that $A^2 = 4A - 7I$ where I is the identity matrix. Express A^4 in the form $pA + qI$. [4]

3. (4 marks)

Determine two numbers which have a sum of 3 and a product of 3.

4. (8 marks)

(a) Prove, by contradiction, $\log_{10} 2$ is irrational. [4]

(b) Prove, by exhaustion, $(n+1)^3 \geq 3^n$ where n is a counting number ≤ 4. [4]

5. (5 marks)

Prove, by mathematical induction, that $n^3 + 2n$ is divisible by 3 for any positive integer n .

6. (7 marks)

Simplify the following complex expressions leaving the answer in the form a + bi.

(a) $6-7i-(2-4i)$ [2]

(b) $\dfrac{4+3i}{1-2i}$ [3]

(c) $\dfrac{-i}{i^3}$ [2]

7. (8 marks)

(a) One root of the equation $z^2 + az + b = 0$, where a and b are real constants, is $4 - i$. Determine the value of a and b. [4]

(b) Solve the equation $3z = (7 + 2i)^2 - \bar{z}$ for the complex number z. (Hint: let $z = a + bi$) [4]

8. (4 marks)

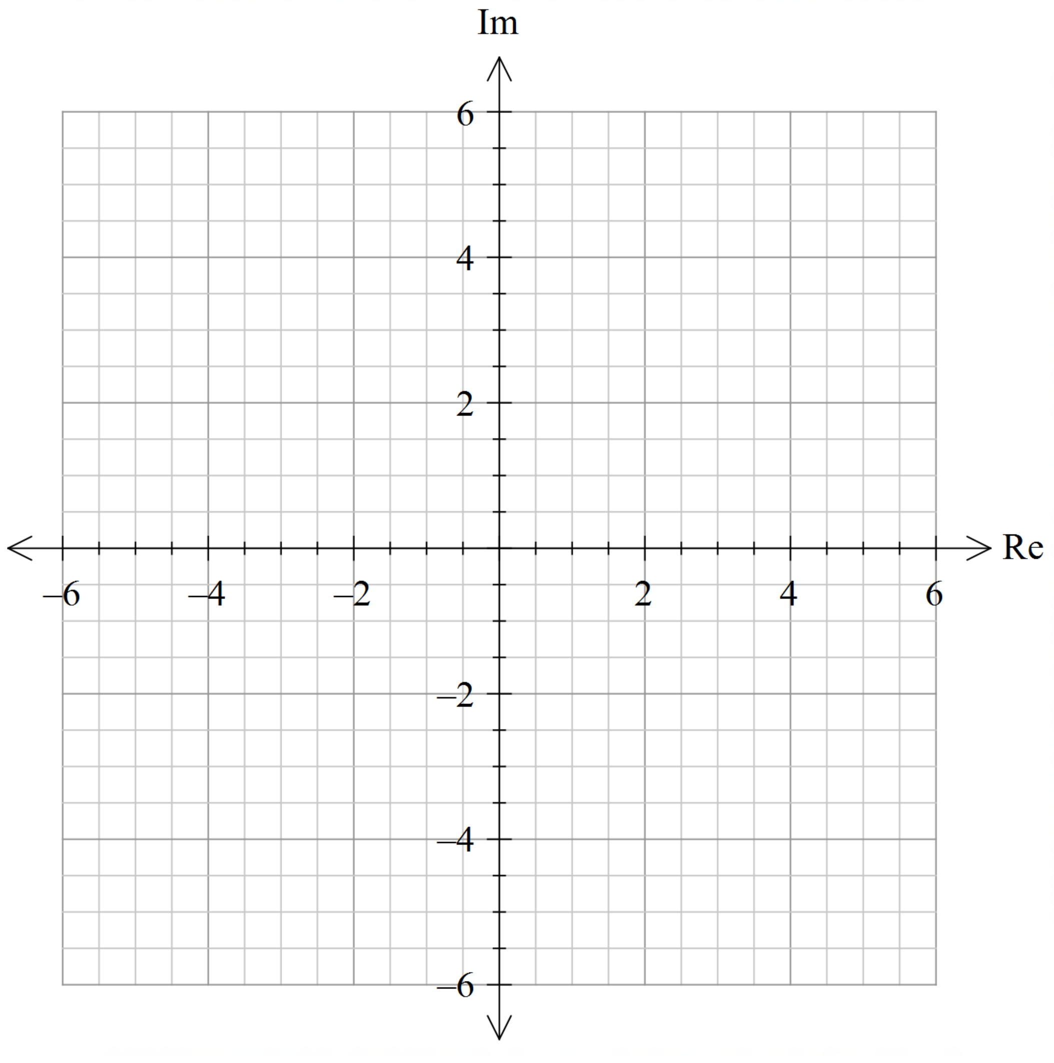
Given $z = 3 - 4i$, draw each of the following on the Argand diagram below. Clearly label each answer.

(a) $\bar{z}$ [1]

(b) $i^3 z$ [1]

(c) Im(z) [1]

(d) $i\,\text{Re}(z)$ [1]



9. (7 marks)

Use mathematical induction to prove the following conjecture:

$1 + (1 + x) + (1 + x)^2 + \ldots + (1 + x)^{n-1} = \frac{(1+x)^n - 1}{x}$, $n \geq 1$, n a counting number.